EE 313 – Linear Signals and Systems
Midterm I – July 1, 2016
(closed book exam)
1:00 PM -2:15 PM

Name:___________________________________
UTEID:___________________________________

Please:

- Note that the exam is individual work.
- Place your ID on the table.
- Make sure you show your work.
- Note attached equation sheet.

1. (15 pts) (Complex Numbers)
   a. Show that magnitude of complex exponential $|e^{j\theta t}| = 1$ for any θ and any t.

   b. Consider the set of complex numbers z given by $|z| < 3$. Plot the set in the complex plane.

   c. Given the complex number $z_1 = -4+3j$, find the polar representation forms for the roots $z_1^{1/4}$

d. Assume we have the complex numbers A and z, where they can be represented in polar form as A = a $e^{j\theta}$, and z = b $e^{j\beta}$. Assume that their conjugates are represented by A* and z* respectively. Show that $Az^n + A^*(z^*)^n$ cane be written as:

$$Az^n + A^*(z^*)^n = x[n]cos(\beta n + \theta)$$

Also show the expression for x[n].

2. (25 pts) Assume a continuous-time system is represented by the following differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = 2\frac{dx}{dt} + x(t)$, answer the following questions:

a. Find the characteristic roots for the system.

b. Find the zero-input response for the system. Assume the initial conditions y(0) = 0, and y'(0) = 1.

c. Is the system asymptotically stable? Justify your answers.

d. Determine the impulse response of the system.

e. Is the system causal? Justify your answer.

2. (20 pts) Assume a discrete-time system is represented by the following difference equation $y[n+2] + 5y[n+1] + 6y[n] = 2x[n+1] + x[n]$, answer the following questions with initial conditions y[-2] = 0, y[-1]=1.

   a. Find the characteristic roots for the system.

b. Find the zero-input response.

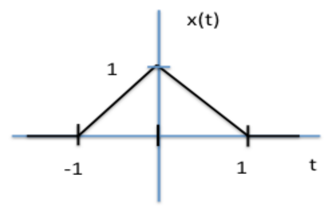
c. Is the system asymptotically stable? Justify your answers.

d. Find the impulse response.

3. (20 pts) (Convolution with continuous-time signals)

   a. Determine the zero-state output of a system represented by the impulse response $h(t) = \frac{1}{t+1}u(t)$, and the input to the system is the step function $x(t) = 4u(t)$.

b. Assume the triangular wave shown below is fed to a system represented by the impulse response $h(t) = \sum_{k=-1}^{k=1} \delta(t - \frac{3}{2}k)$, ($k$ takes integer values), derive and plot the zero-state response.



4. (20 pts) (Convolution with discrete-time signals)
   a. Determine the zero-state output of a system represented by the impulse response $\mathrm{h[n]} = (0.2)^n u[n]$,and the input to the system is the function $\mathrm{x[n]} = (5)^n u[n]$.

b. Determine the zero-state output of a system represented by the impulse response $h[n] = n(u[n] - u[n-4])$ and when the input to the system is $x[n] = (u[n] - u[n-4])$.